Introduction to Solar Radiative Transfer:
I Basic Radiative Transfer

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Overview

- I Basic Radiative Transfer (Intensity, emission, absorption, source function, optical depth)

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- II Detailed Radiative Processes (Spectral lines, radiative transitions, collisions, continuum processes, Saha-Boltzmann)

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)

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- **Mihalas:** Stellar Atmospheres

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- **Allen:** Astrophysical Quantities

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- **1842** Becquerel photographs spectra, discovers UV lines beyond the visible
- **1858** Bunsen and Kirchhoff discover wavelength correspondence between bright flame emission and dark solar absorption lines. Start of quantitative spectroscopy.

Why Radiative Transfer?

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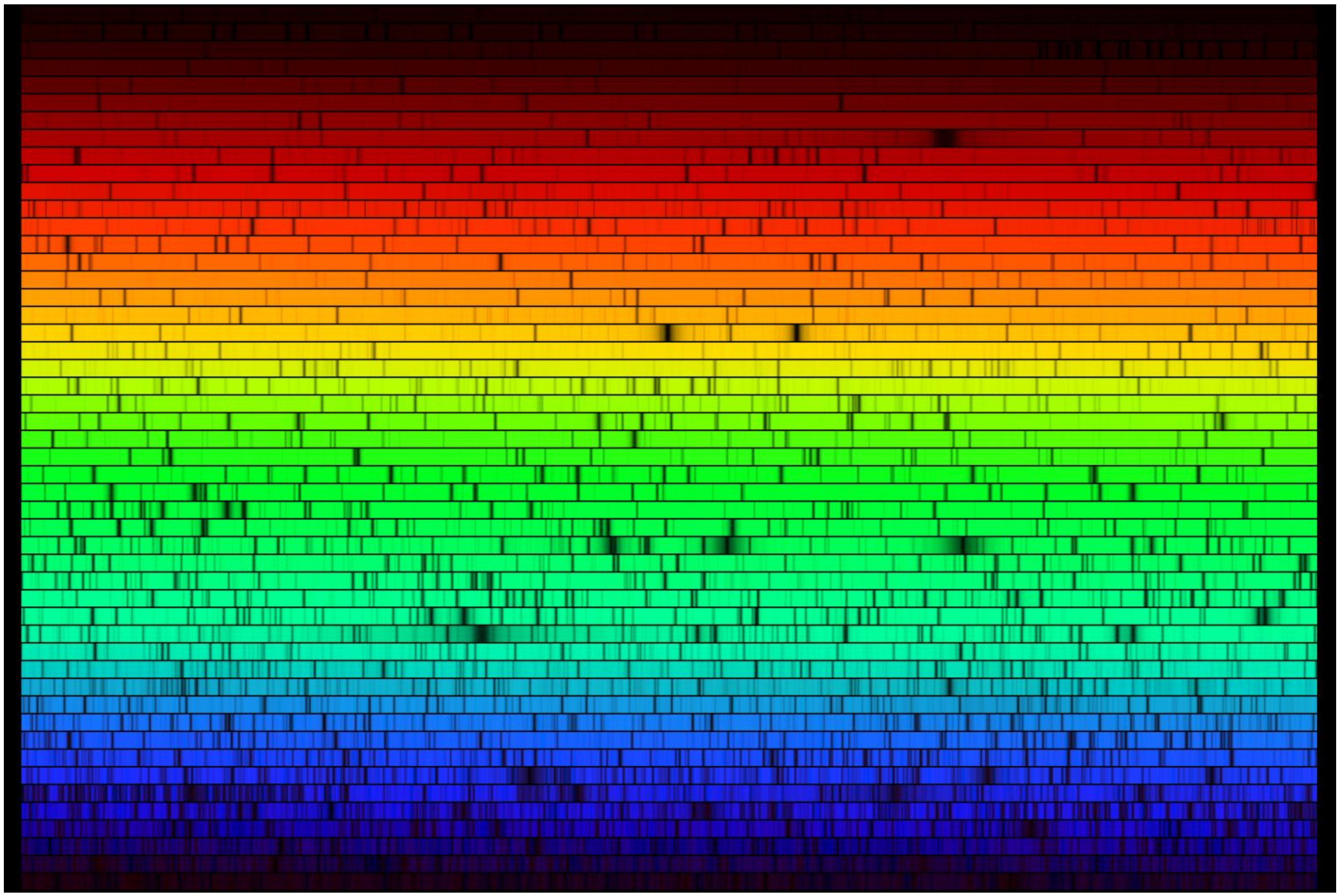
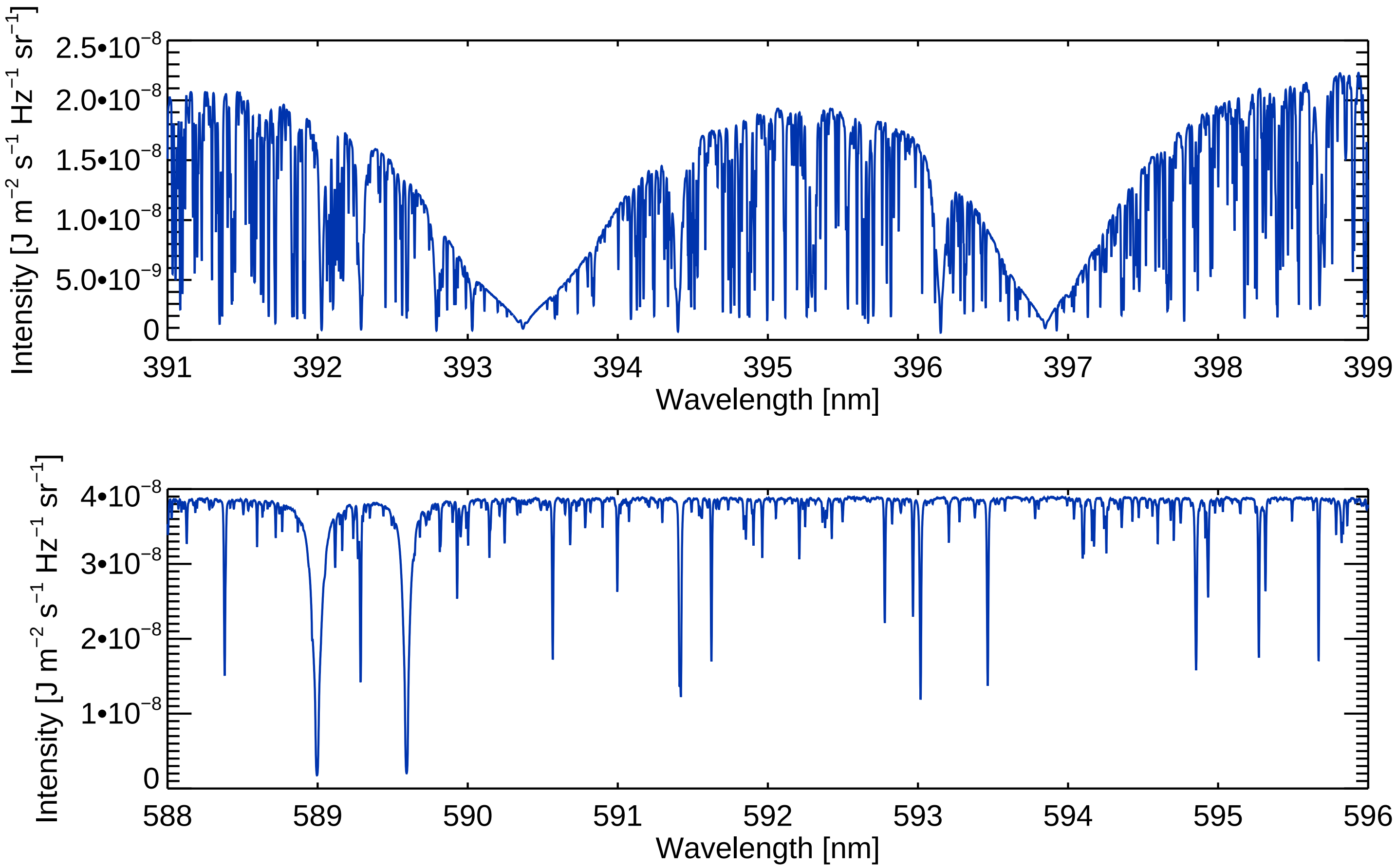
- Observational techniques need to be applied to obtain the relevant properties of the radiation signal
- We need to understand how the radiative signal is modified as it travels from the object to our instruments

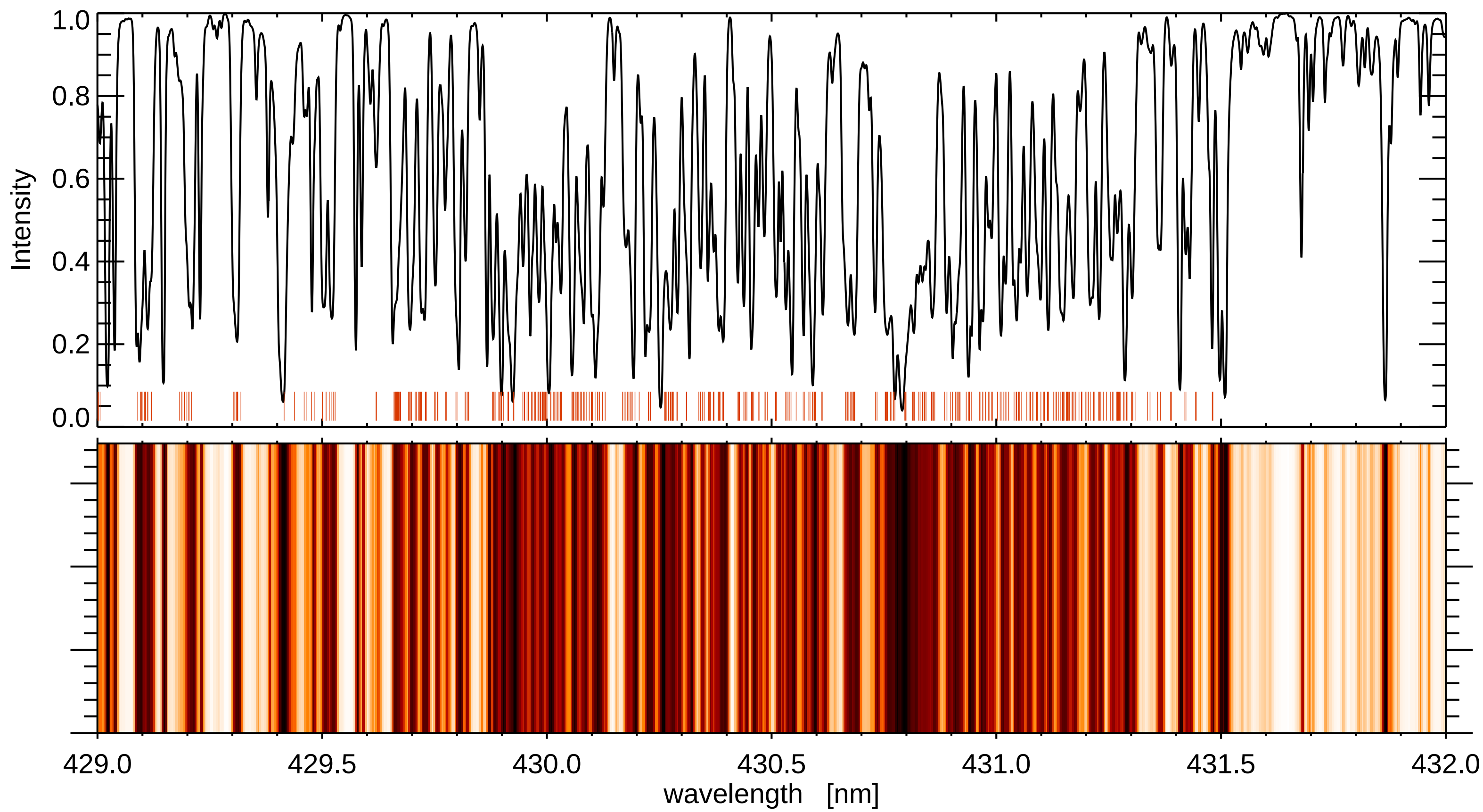
Solar line spectrum



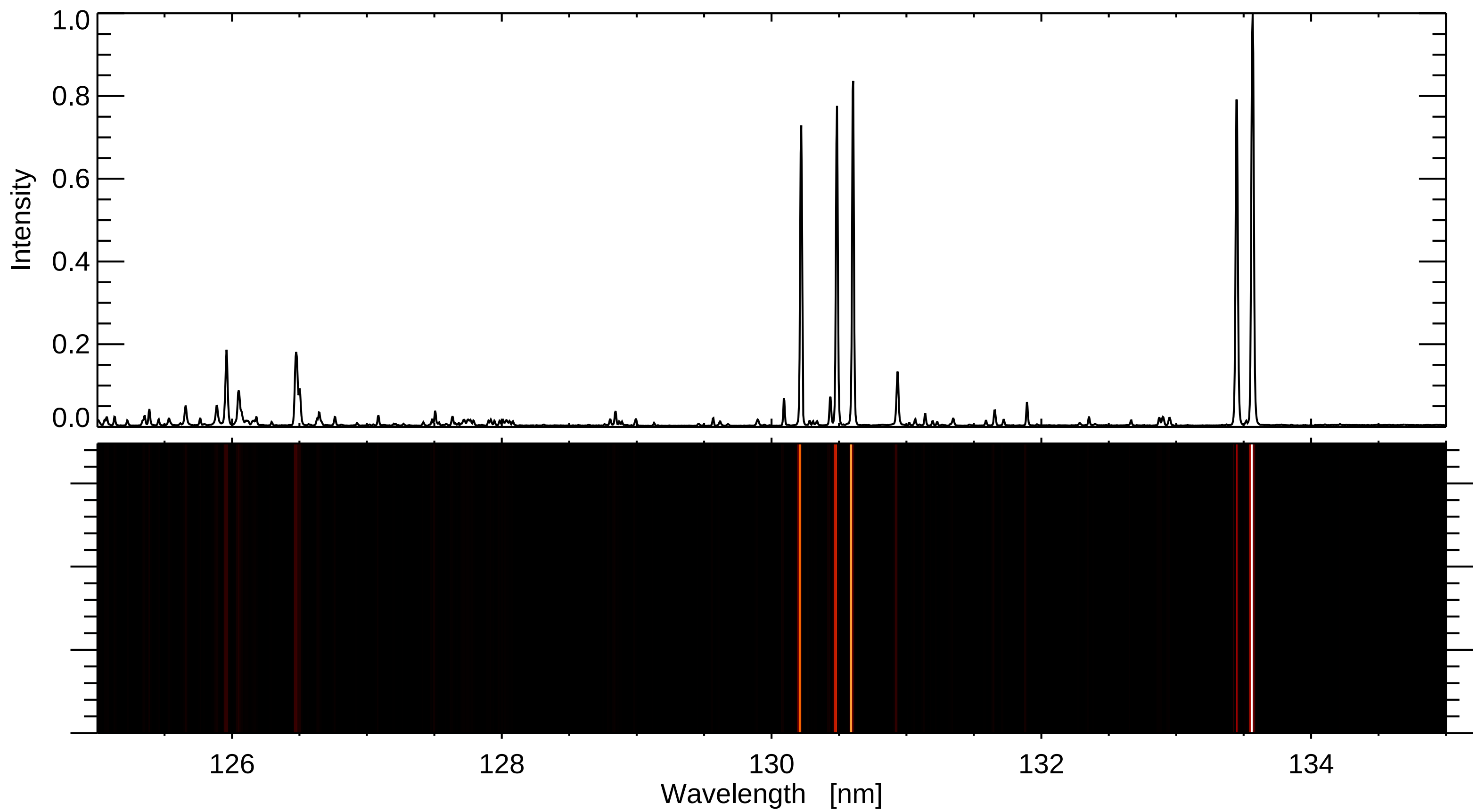
Differences in Spectral Lines



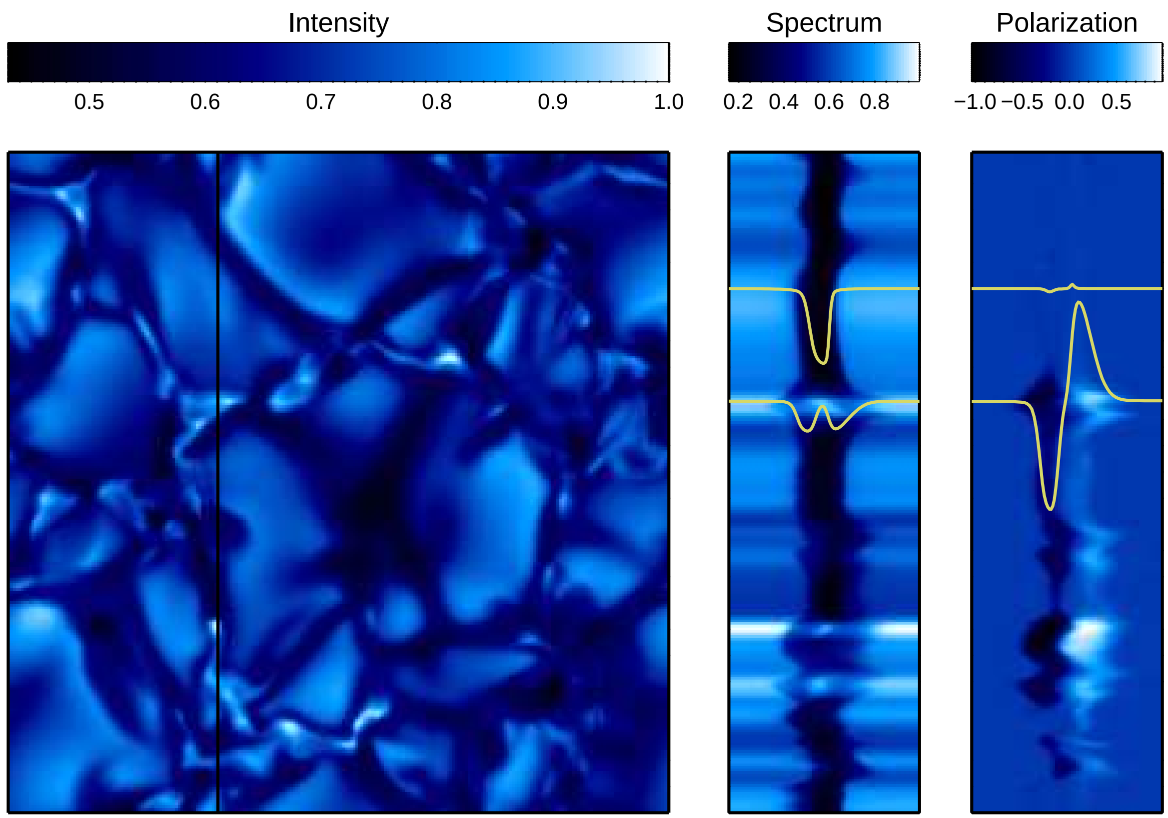
Molecular lines in the solar spectrum



In the UltraViolet the Spectral Lines are in Emission



Spatially Resolved Spectral Lines



Basic Radiative Transfer: Radiation Field

Specific Intensity:

$$\mathrm{d}E_\nu \equiv I_\nu(\vec{r}, \vec{l}, t)\,\mathrm{d}t\,\mathrm{d}A\,\mathrm{d}\nu\,\mathrm{d}\Omega \tag{1}$$
$$= I_\nu(x, y, z, \theta, \varphi, t)\cos\theta\,\mathrm{d}t\,\mathrm{d}A\,\mathrm{d}\nu\,\mathrm{d}\Omega$$

Units: J s^{-1} m^{-2} Hz^{-1} ster^{-1}

Mean intensity:

$$J_\nu(\vec{r}, t) \equiv \frac{1}{4\pi}\int I_\nu\,\mathrm{d}\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\nu \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi \tag{2}$$

Units: J s^{-1} m^{-2} Hz^{-1} ster^{-1}

Basic Radiative Transfer: Radiation Field

Flux:

$$\mathcal{F}_\nu(\vec{r}, \vec{n}, t) \equiv \int I_\nu \cos\theta \, \mathrm{d}\Omega = \int_0^{2\pi} \int_0^{\pi} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \tag{3}$$

Units: J s^{-1} m^{-2} Hz^{-1}

Flux in radial direction:

$$\begin{aligned} \mathcal{F}_\nu(z) &= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi + \int_0^{2\pi} \int_{\frac{\pi}{2}}^{\pi} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \qquad (4) \\ &= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi - \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu(\pi - \theta) \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \\ &\equiv \mathcal{F}_\nu^+(z) - \mathcal{F}_\nu^-(z) \end{aligned}$$

Basic Radiative Transfer: Local Changes

Emission:

$$
\begin{aligned}
\mathrm{d}E_\nu &\equiv j_\nu \,\mathrm{d}V \,\mathrm{d}t \,\mathrm{d}\nu \,\mathrm{d}\Omega \\
\mathrm{d}I_\nu &= j_\nu(s)\mathrm{d}s
\end{aligned}
\tag{5}
$$

Units: J m^{-3} s^{-1} Hz^{-1} ster^{-1}

Extinction:

$$
\mathrm{d}I_\nu \equiv -\alpha_\nu I_\nu \mathrm{d}s \tag{6}
$$

Units: m^{-1}

Basic Radiative Transfer: Local Changes

Source function:

$$S_\nu = j_\nu / \alpha_\nu \tag{7}$$

Units: J s^{-1} m^{-2} Hz^{-1} $ster^{-1}$

$$S_\nu^{\rm tot} = \sum j_\nu / \sum \alpha_\nu \tag{8}$$

$$S_\nu^{\rm tot} = \frac{j_\nu^c + j_\nu^l}{\alpha_\nu^c + \alpha_\nu^l} = \frac{S_\nu^c + \eta_\nu S_\nu^l}{1 + \eta_\nu}, \quad \eta_\nu \equiv \alpha_\nu^l / \alpha_\nu^c \tag{9}$$

Basic Radiative Transfer: Transport Equation

Transport along a ray:

$$\mathrm{d}I_\nu(s) = I_\nu(s + \mathrm{d}s) - I_\nu(s) = j_\nu(s)\mathrm{d}s - \alpha_\nu(s) I_\nu(s)\mathrm{d}s \tag{10}$$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha_\nu I_\nu$$

$$\frac{\mathrm{d}I_\nu}{\alpha_\nu \mathrm{d}s} = S_\nu - I_\nu$$

Optical length and thickness:

$$\mathrm{d}\tau_\nu \equiv \alpha_\nu(s)\mathrm{d}s \tag{11}$$

$$\tau_\nu(D) = \int_0^D \alpha_\nu(s)\mathrm{d}s$$

Basic Radiative Transfer: Transport Equation

Transport along a ray:

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu \tag{12}$$

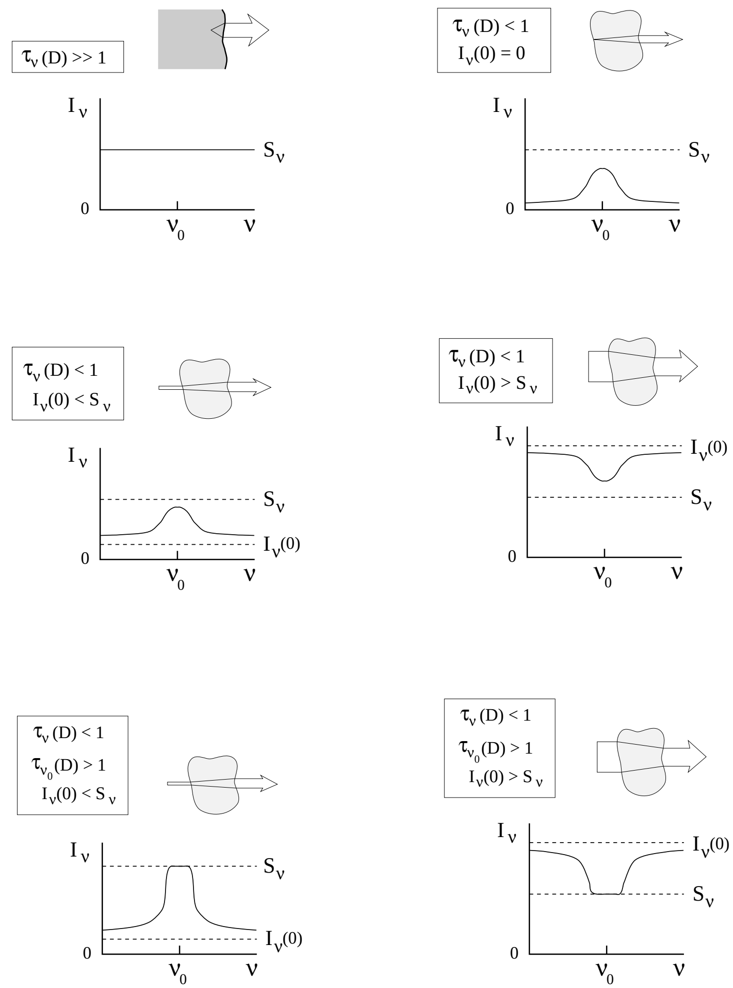
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t)e^{-(\tau_\nu - t)}\mathrm{d}t$$

Homogeneous medium:

$$I_\nu(D) = I_\nu(0)e^{-\tau_\nu(D)} + S_\nu\left(1 - e^{-\tau_\nu(D)}\right) \tag{13}$$

Optically thick: $I_\nu(D) \approx S_\nu$
Optically thin: $I_\nu(D) \approx I_\nu(0) + \left[S_\nu - I_\nu(0)\right]\tau_\nu(D)$

Basic Radiative Transfer: Homogeneous Medium



Basic Radiative Transfer: Through an Atmosphere

Optical depth:

$$d\tau_{\mu\nu} = \alpha_\nu ds \equiv -\alpha_\nu \frac{dz}{|\mu|} \tag{14}$$

$$\tau_\nu(z_1) = \int_{z_{\rm surf}}^{z_1} -\alpha_\nu dz = \int_{z_1}^{z_{\rm surf}} \alpha_\nu dz$$

Standard plane parallel transport equation:

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \tag{15}$$

Basic Radiative Transfer: Through an Atmosphere

Formal solution in upward direction:

$$I_\nu^+(\tau_\nu, \mu) = \int_{\tau_\nu}^{\infty} S_\nu(t) e^{-(t-\tau_\nu)/\mu} \mathrm{d}t/\mu, \quad \mu > 0 \tag{16}$$

Formal solution in downward direction:

$$I_\nu^-(\tau_\nu, \mu) = -\int_{0}^{\tau_\nu} S_\nu(t) e^{-(t-\tau_\nu)/\mu} \mathrm{d}t/\mu, \quad \mu < 0 \tag{17}$$

Basic Radiative Transfer: Eddington–Barbier

Emergent intensity at the surface:

$$I_\nu^+(\tau_\nu = 0, \mu) = \int_0^\infty S_\nu(t) e^{-t/\mu} \mathrm{d}t/\mu \tag{18}$$

Substitute power series:

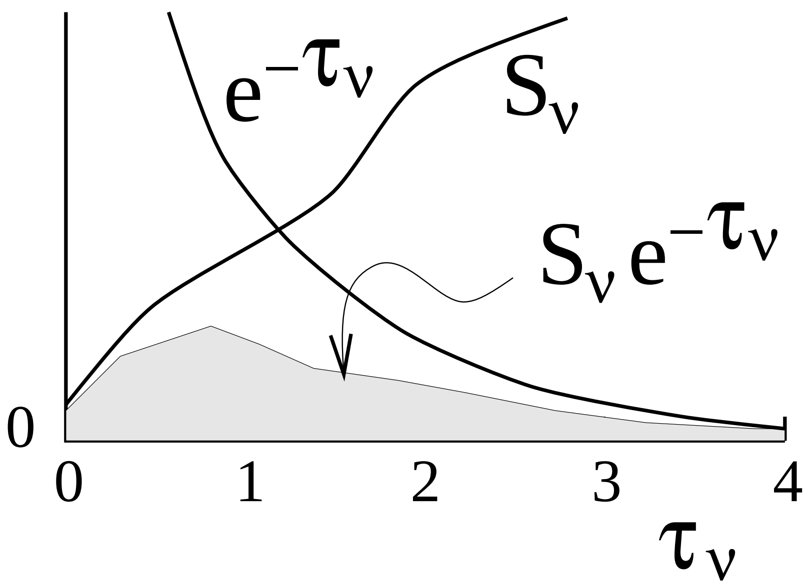
$$S_\nu(\tau_\nu) = \sum_{n=0}^{N} a_n \tau_\nu^n \tag{19}$$

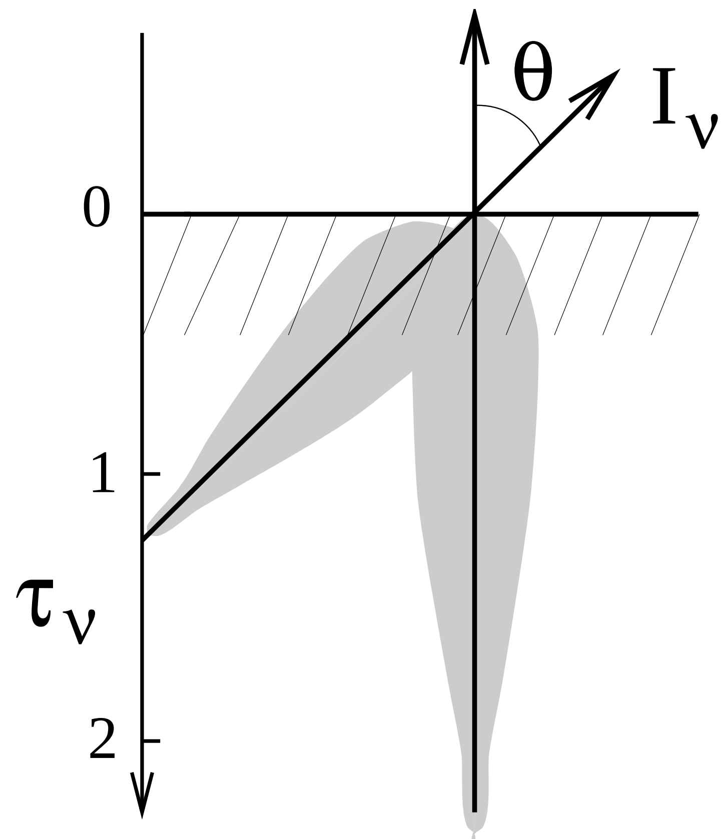
$$I_\nu^+(\tau_\nu = 0, \mu) = a_0 + a_1\mu + 2a_2\mu^2 + \ldots + n! a_N \mu^N$$

Eddington–Barbier relation:

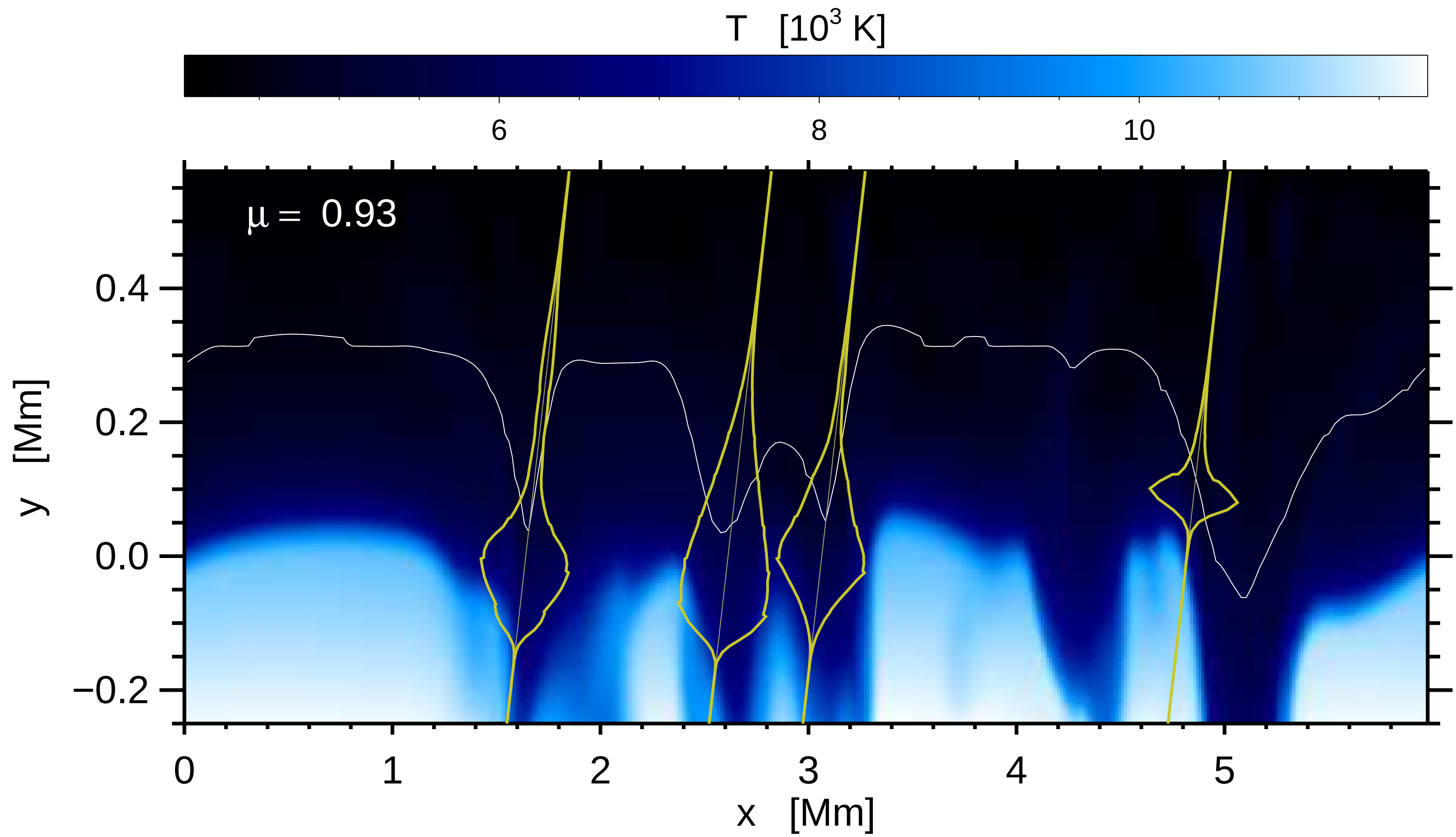
$$I_\nu^+(\tau_\nu = 0, \mu) \approx S_\nu(\tau_\nu = \mu) \tag{20}$$

Basic Radiative Transfer: Eddington–Barbier

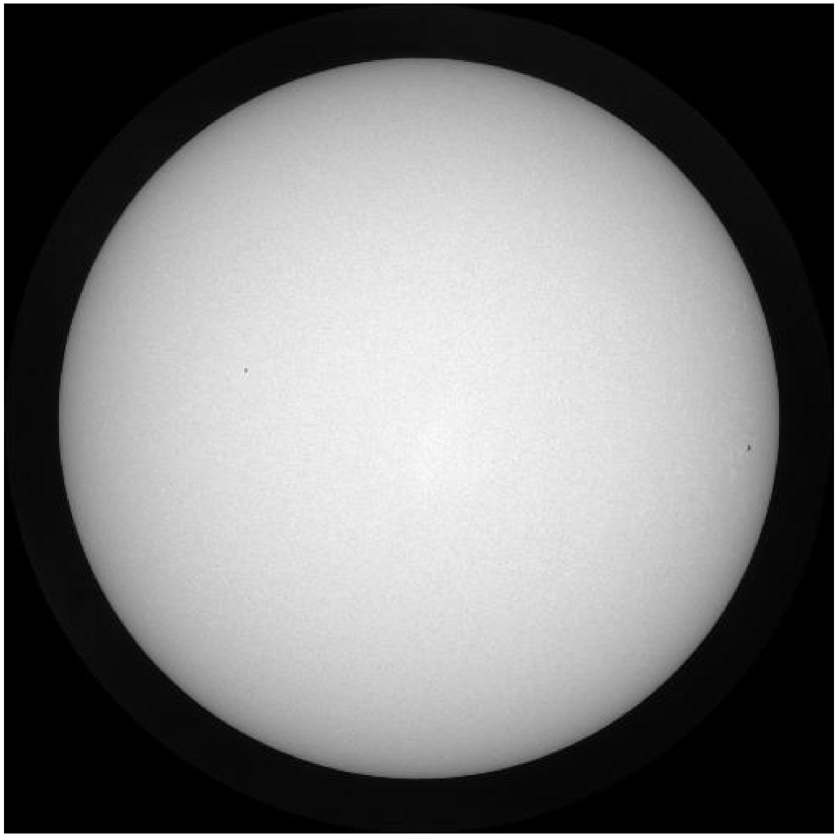


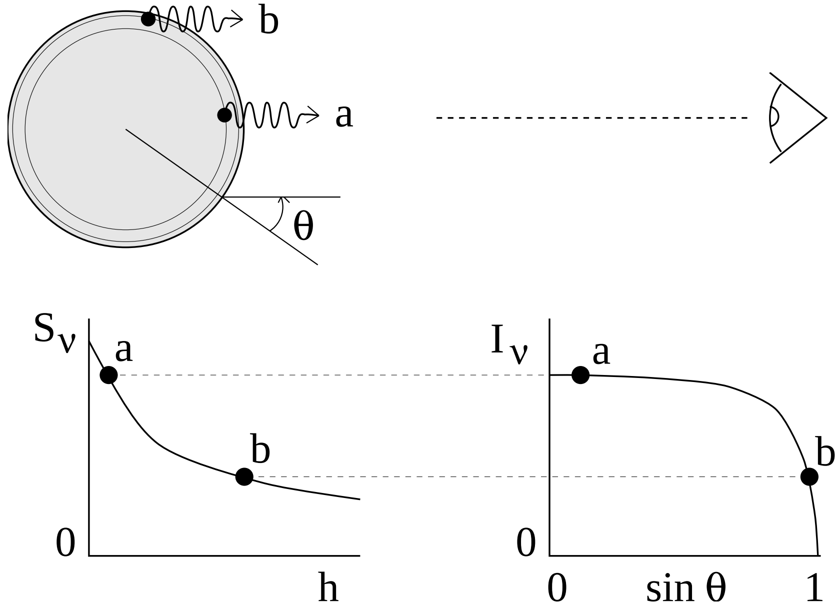


Basic Radiative Transfer: Eddington–Barbier



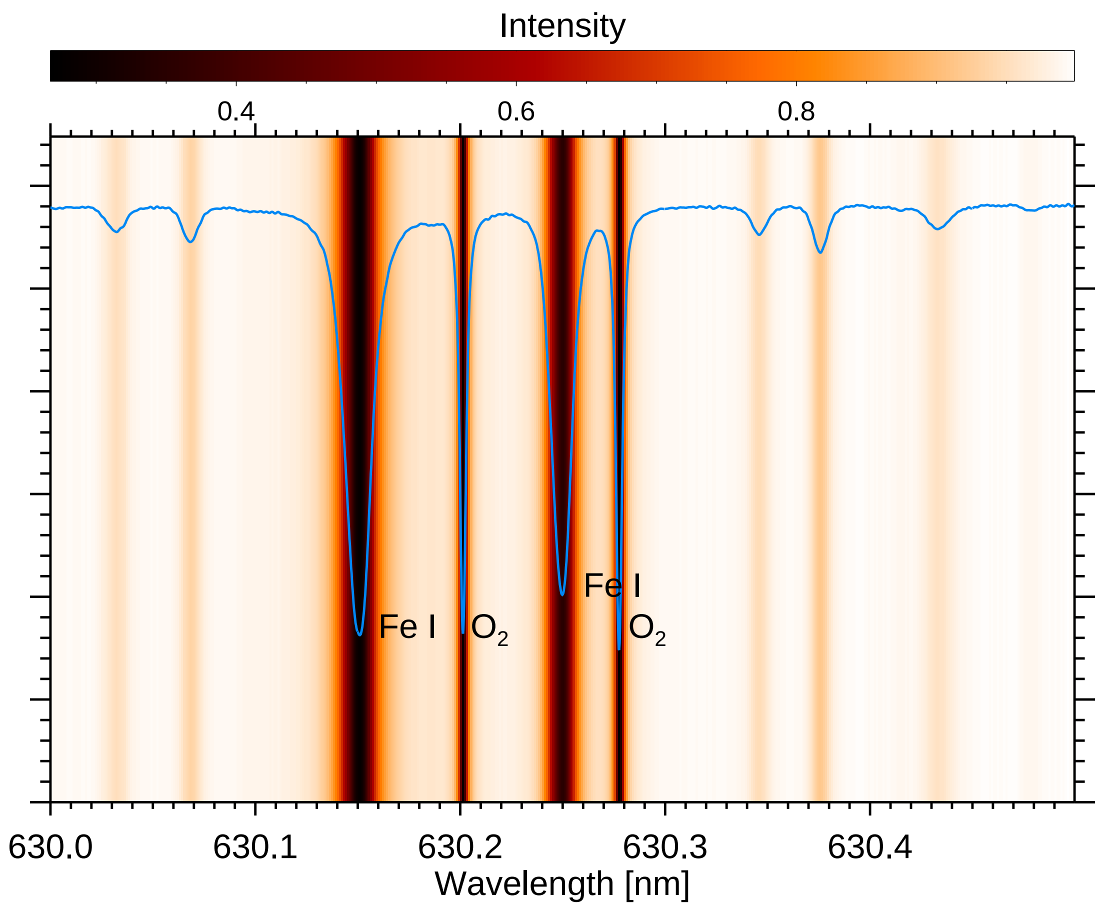
Basic Radiative Transfer: Limb Darkening





End Part I

Molecular Oxygen in the Earth Atmosphere



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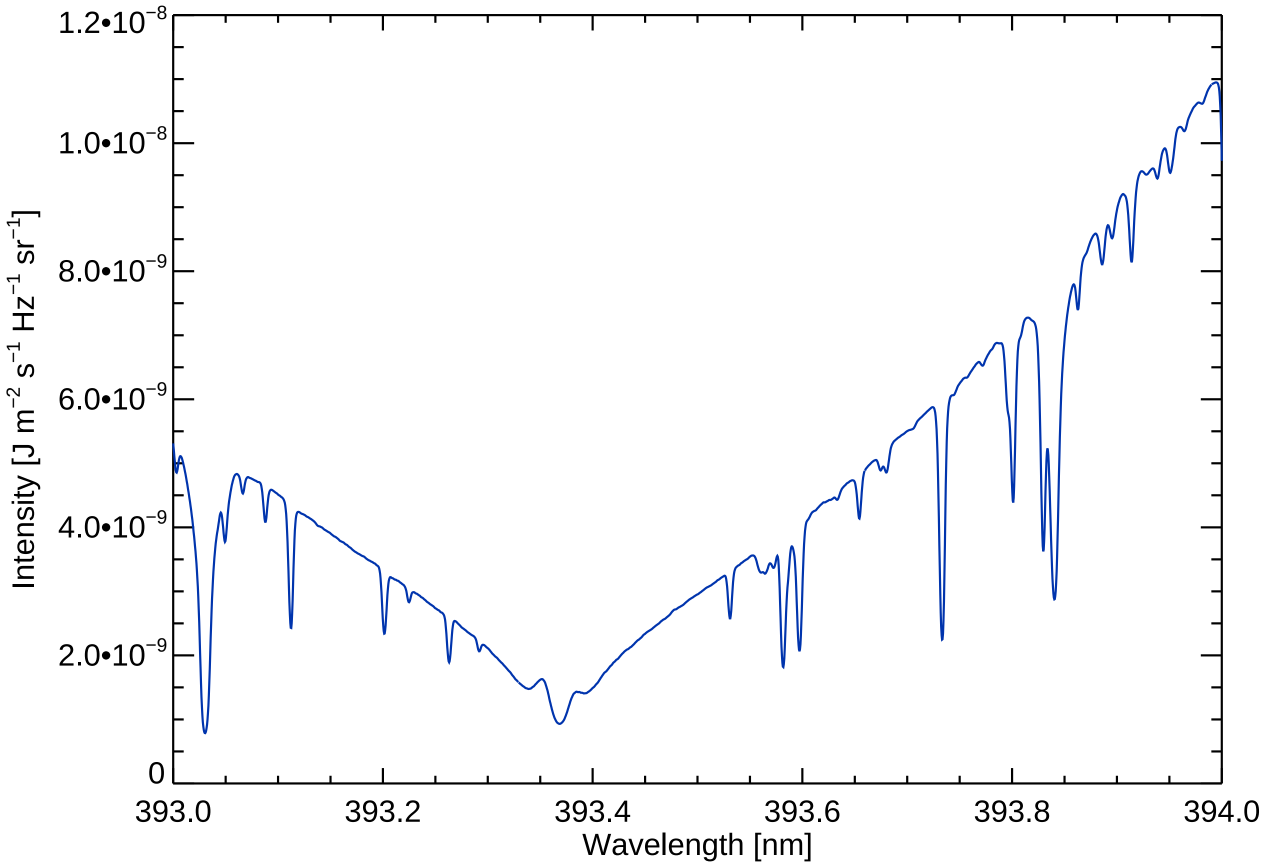
Differences in spectral lines



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Invariance of Specific Intensity along Rays

Specific Intensity has been defined in such a way as to be independent of the source and the observer.

$$\mathrm{d}E_\nu = I_\nu \cos\theta \,\mathrm{d}t\,\mathrm{d}A\,\mathrm{d}\nu\,\mathrm{d}\Omega = I'_\nu \cos\theta' \,\mathrm{d}t\,\mathrm{d}A'\,\mathrm{d}\nu\,\mathrm{d}\Omega'$$

$$\mathrm{d}\Omega = \mathrm{d}A' \,\cos\theta' / R^2$$

$$\mathrm{d}\Omega' = \mathrm{d}A \,\cos\theta / R^2$$

$$I_\nu = I'_\nu$$

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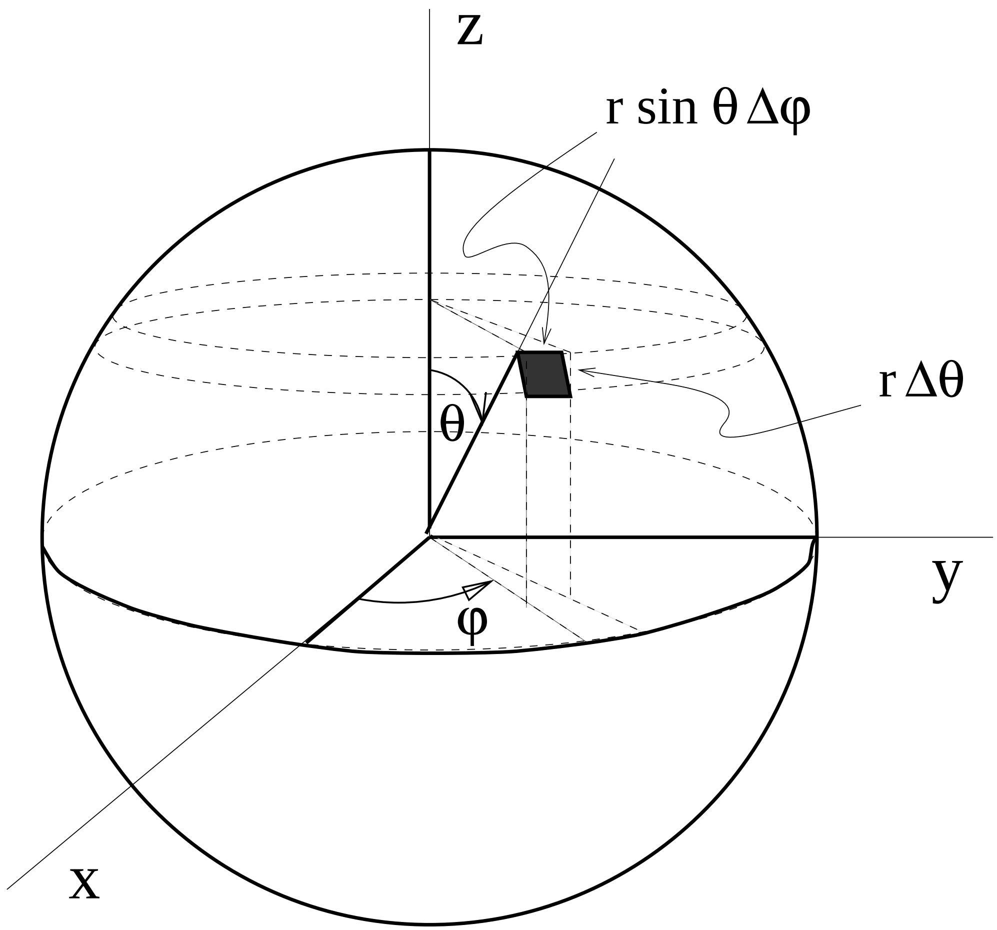
Spherical Coordinates



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