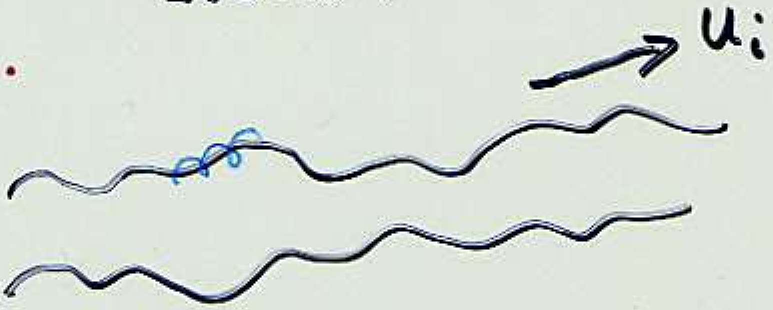
Lecture #2



$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} K_{ij} \frac{\partial f}{\partial x_j} - U_i \frac{\partial f}{\partial x_i} + \frac{1}{3} \frac{\partial U_i}{\partial x_i} \frac{\partial f}{\partial \ln p}$$

$$S_i = -K_{ij} \frac{\partial f}{\partial x_j} - \frac{1}{3} U_i \frac{\partial f}{\partial \ln p}$$ = streaming flux

$$\delta_i = \frac{3 S_i}{w f}$$

$K_{\parallel}$ parallel diffusion

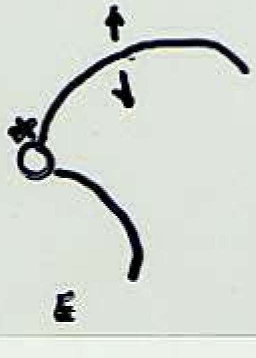
$\rightarrow \frac{1}{3} \lambda w$

$$K_{ij} = K_{\perp} \delta_{ij} + (K_{\perp} - K_{\parallel}) \frac{B_i B_j}{B^2} + \epsilon_{ijk} \frac{B_k}{|B|} K_A$$

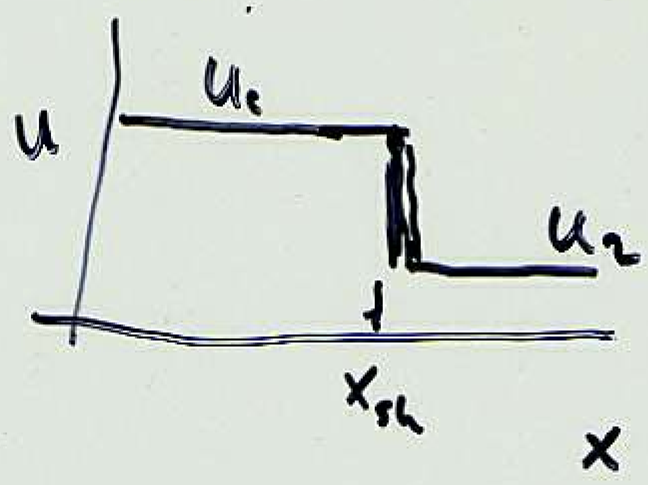
$K_{\perp}$ perpendicular diffusion

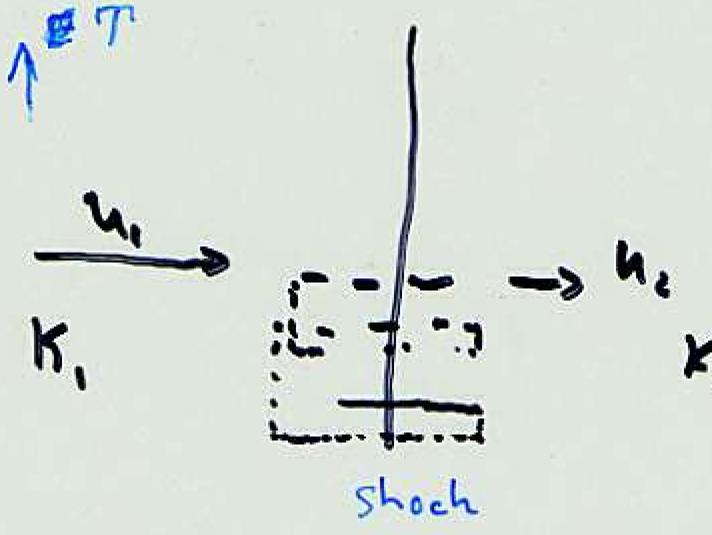
$K_{\perp} \simeq .1 K_{\parallel}$

$K_A = \frac{1}{3} r_g w$



Apply transport eqn to a shock.





$$(\Delta T)_{2\text{ trips}} \propto (u_2 - u_1)$$

Parker eqn

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial f}{\partial x}\right) - u\frac{\partial f}{\partial x} + \left(\frac{\partial u}{\partial x}\right)\frac{1}{3}\frac{\partial f}{\partial \ln p}$$

$\partial f / \partial t = 0$

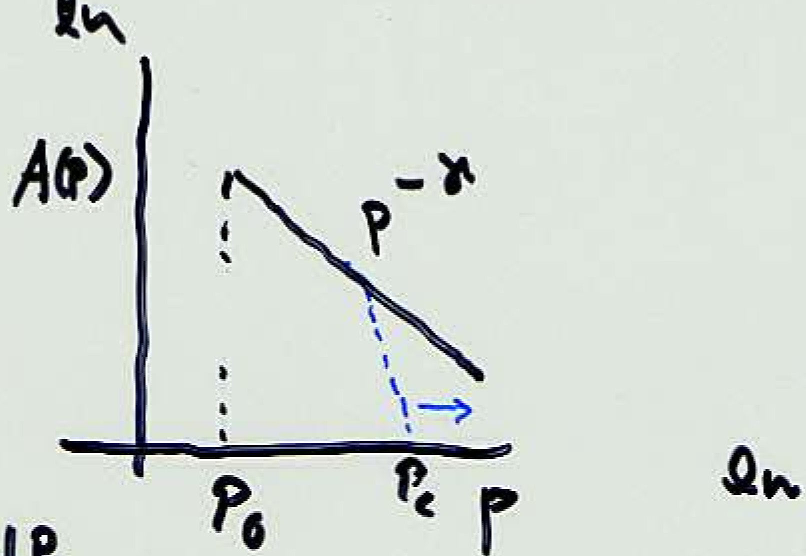
solution

$$f(x, p) = \begin{cases} A(p) e^{u_1 x / \kappa_1} & x < x_{sh} \\ A(p) & x > x_{sh} \end{cases}$$

$$p \frac{dA(p)}{dp} = \frac{3 u_1}{u_2 - u_1} A(p)$$

$$A(p) = A_0 \left(\frac{p}{p_0}\right)^{-\gamma} \qquad \gamma = \frac{3r}{r-1}$$

$$r = \frac{u_1}{u_2}$$



$$\tau_{ACC}^{-1} \sim \frac{u_1^2}{\kappa_1} \approx \frac{1}{p_c}\frac{dp_c}{dt}$$

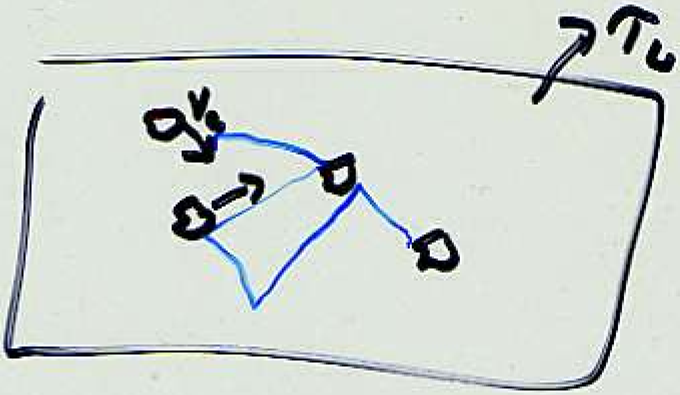
if $r \Rightarrow 4$

$\gamma \Rightarrow 4$

$$\frac{dj}{dp} \sim p^{-2}$$

Other mechanisms

2nd-order Fermi



$$\Rightarrow \frac{dp}{dt} \sim \frac{V_c^2}{c^2}\frac{p}{\tau_{sc}}$$

$$\tau_{sc} = \frac{\lambda_{sc}}{w}$$

Syrovatsky : 1. not a power law in general.

2. index of power law involves all of the parameters

$V_c \Rightarrow V_A$

Solar Energetic Particles (SEP’s)

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LPL, University of Arizona

Lecture 2

High-Energy Charged Particles:
Topics to be covered in 2 lectures

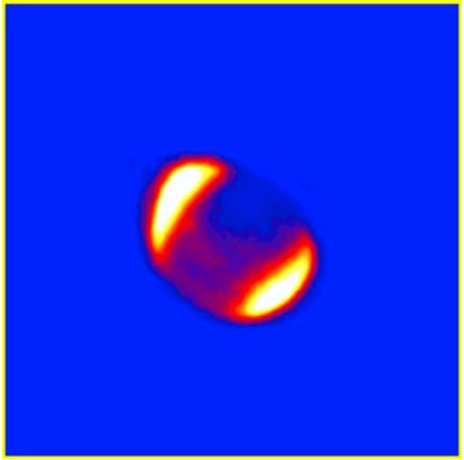
- Lecture 1:
  - Overview of energetic particles in the solar system
  - Basic theory of energetic particles 1
    - Particle distributions, diffusion, convection

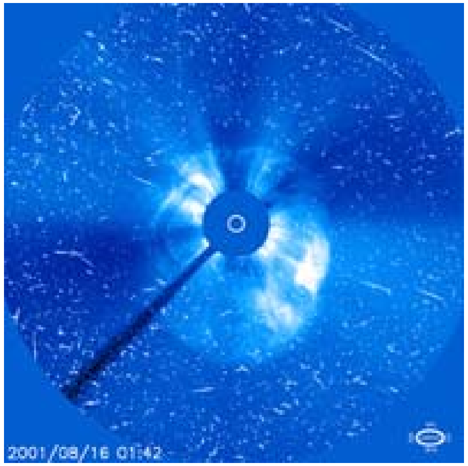
- Lecture 2:
  - Basic theory 2: Acceleration Mechanisms
    - Shock acceleration (CMEs and flares)
    - Stochastic acceleration (flares?)
  - Non-diffusive treatment

Evidence for shock acceleration

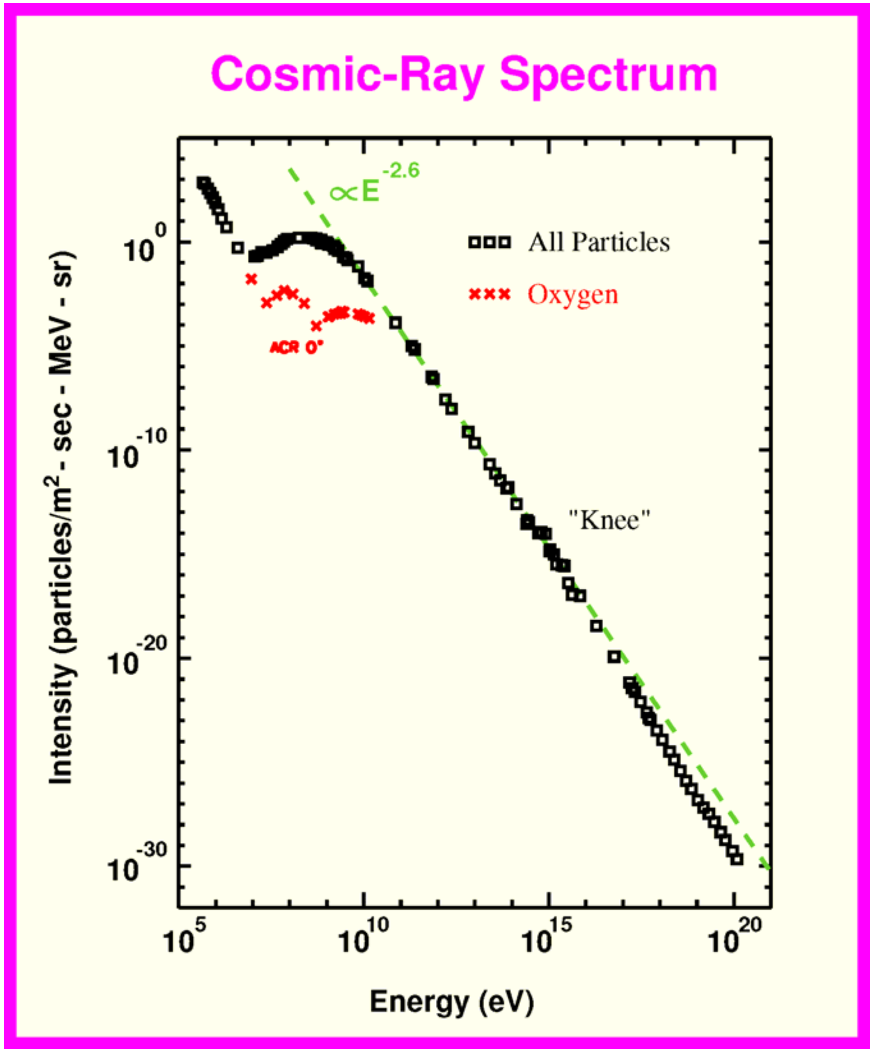
- Indirect evidence:
  - Energetic particles in space share one common characteristic:
    - energy spectra are often ***Power Laws***
    - Diffusive shock acceleration theory ***naturally*** explains this
    - spectral exponents should vary little from one event to the next.

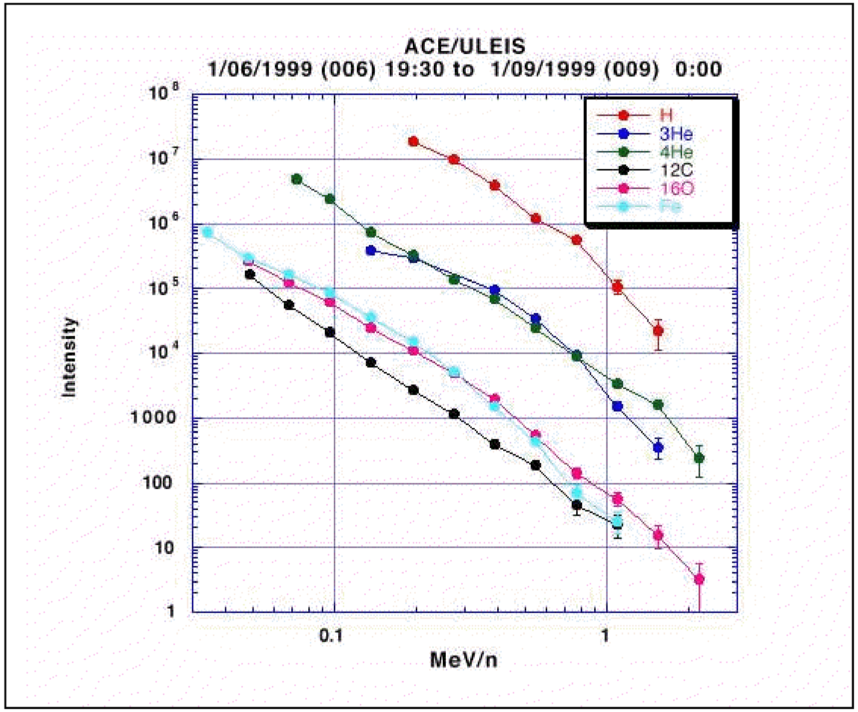
- Direct evidence:
  - Numerous observations of energetic particles associated with shocks
    - Observations of shocks with no accelerated particles too. This is not well understood.





Observed Power-law spectra

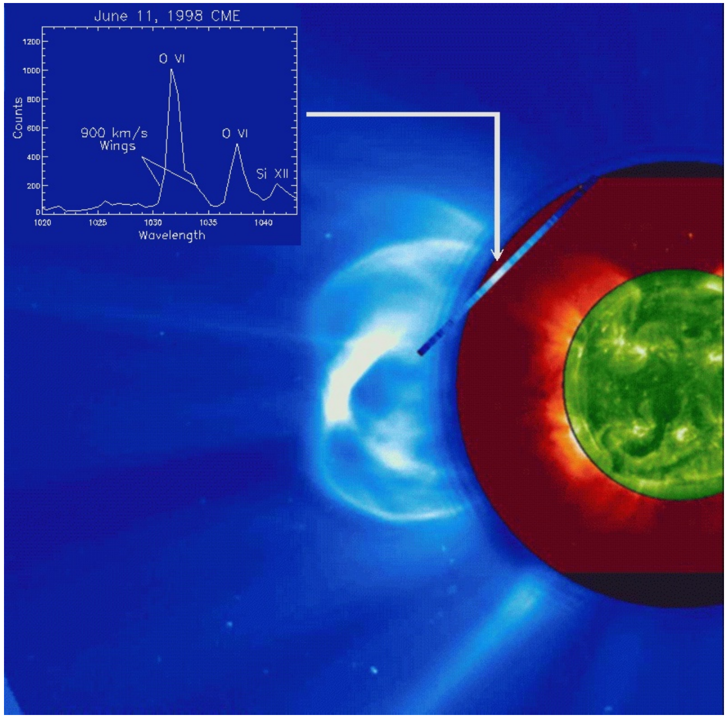




Mason et al., 1999

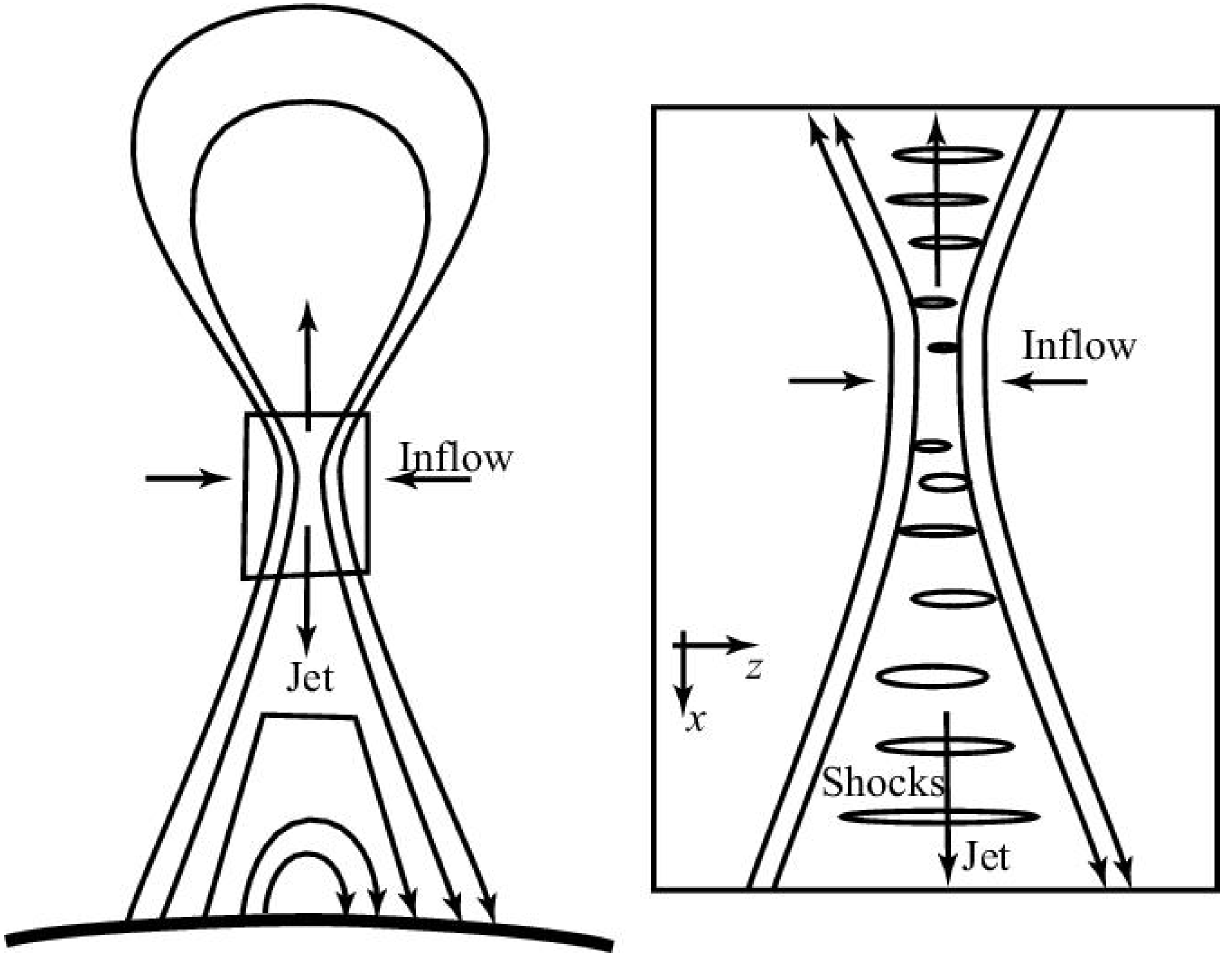
Where do shocks exist?

- Direct observations of collisionless shocks have been made since the first observations of the solar wind by Mariner 2.
- The Earth's bow shock has been crossed thousands of times
- Theoretically, we expect shocks to form quite easily.
    - In the solar corona, shocks can form even when the driver gas is moving <u>slower than the characteristic wave speed</u>.



(Raymond et al., GRL, 27, 1493, 2000)

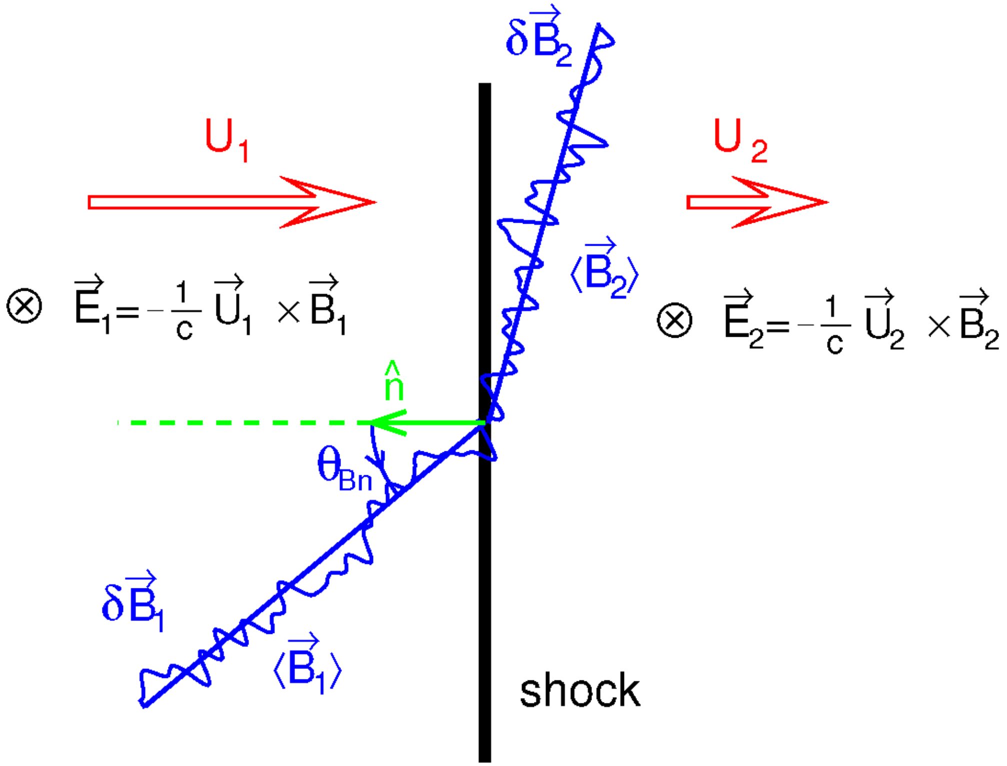
Magnetic Reconnection



Tanuma and Shibata, ApJ, 628, L77, 2005

Diffusive Shock Acceleration

- Solve Parker's transport equation for the following geometry



U and κ change discontinuously across the shock

- The steady-state solution for $f(x,p)$, for an infinite system, is given by

Kennel et al, 1986

$$f(x,p) = \begin{cases} f_0\left(\frac{p}{p_0}\right)^{-\gamma} \exp\left(-\frac{U_1|x|}{\kappa_{xx,1}(p)}\right) & x < 0 \\ f_0\left(\frac{p}{p_0}\right)^{-\gamma} & x \geq 0 \end{cases}$$

where $\gamma = 3U_1/(U_1 - U_2)$

The downstream distribution is power law with a spectral index that depends only on the shock compression ratio!

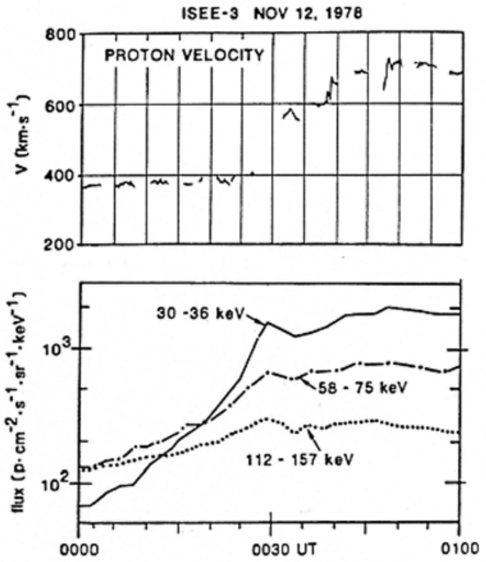
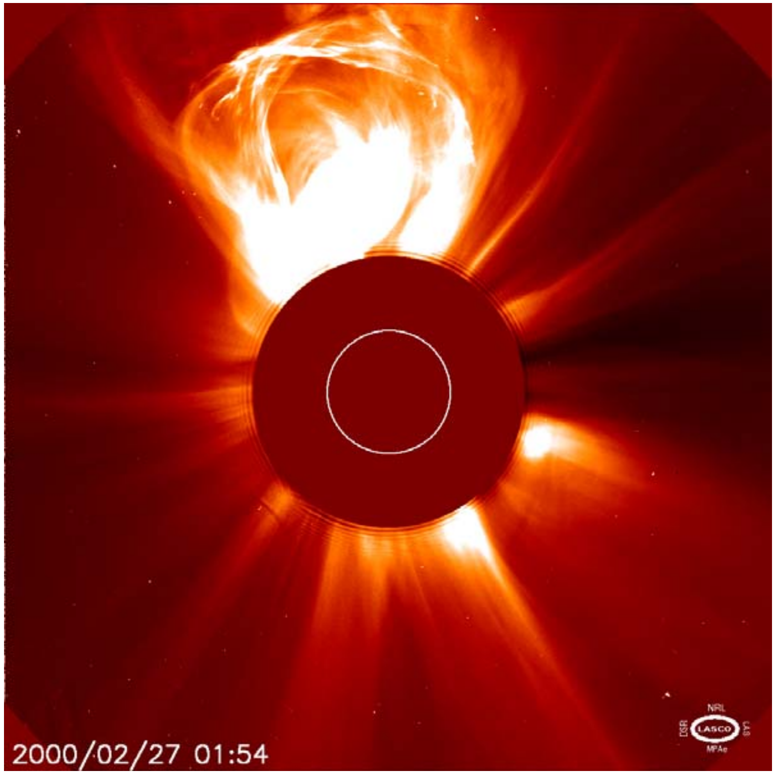
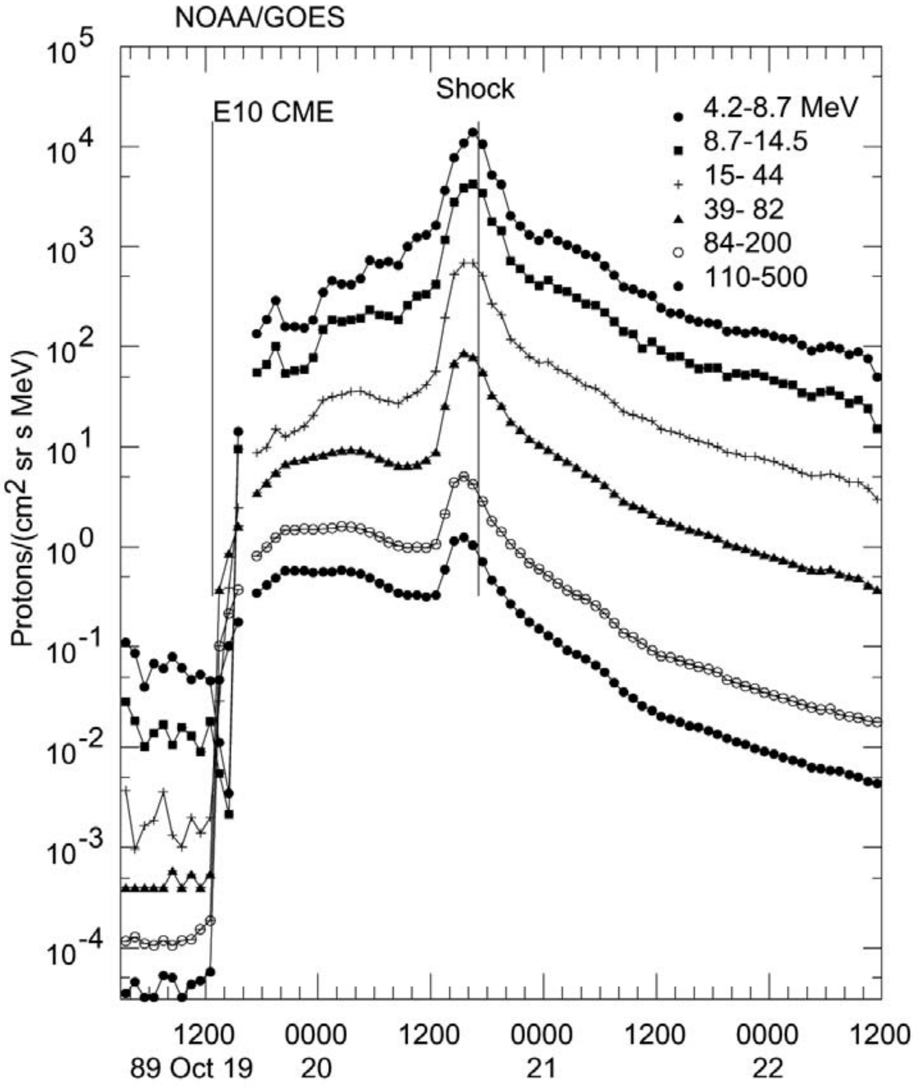


Fig. 1. Solar wind flow speed and energetic protons. The top panel shows the solar wind speed measured by the ISEE-3 solar wind plasma instrument [*Bame et al.*, 1978] and the bottom panel shows the differential fluxes of 30–36 keV, 58–75 keV, and 112–157 keV protons measured by the ISEE-3 nuclear and ionic charge distribution Experiment [*Hovestadt et al.*, 1978]. The period 0000–0100 UT on November 12, 1978, includes the passage of the interplanetary shock over ISEE-3 at 0028:16 UT. The solar wind proton bulk velocity increased slightly, from 380 km s^{-1} to 400 s^{-1}, upstream of the shock and increased to 571 km s^{-1} at the first downstream measurement. The energetic proton fluxes increased roughly exponentially ahead of the shock, with a scale length that increased with increasing energy. The fluxes maximized at the shock, and remained approximately constant downstream of the shock.

Large CME-related SEP events





Reames.SSR, 1999

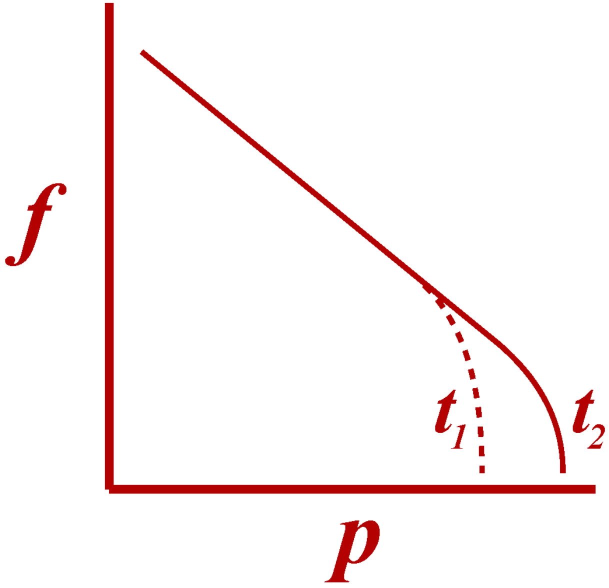
2 related questions:

What is the maximum energy ?

How rapidly can the particles be accelerated?

Spectral cutoffs and rollovers

- Finite acceleration time
  - Parallel shocks → slow
  - Perpendicular shocks → fast
- Free-escape losses
- Limits imposed by the size of the system



All lead to spectral variations that depend on the transport parameters (e.g. species, magnetic turbulence, etc.)

will cause abundance variations that depend on species, and vary with energy

Acceleration Time in Diffusive Shock Acceleration

- The acceleration rate is given by:

$$\frac{1}{\tau} \sim \frac{V_{shock}^2}{\kappa}$$

	Acceleration time	**$(B_{rms}/B)^2$**	**Particle source**	**Characteristic Energy**
Termination shock **(100 AU)**	~ year	~0.3-1	IS pickup ions H, He, N, O, Fe (mostly)	~ 200 MeV (total energy)
CIRs **(2-5 AU)**	~ months	~0.5	Pickup ions, solar wind, enhanced C/O	~ 1-10 MeV/nuc
Earth's bow shock **(1 AU)**	~tens of minutes	~1	Pickup ions, solar wind, magnetosphere ions	~ 100-200 keV/nuc.
Large SEPs **(r > 0.01 AU)**	Minutes or less	??	Suprathermals ***** H (mostly), He, and heavy ions, even M > 50	~1 MeV/nuc, sometimes up to ~20 GeV
Transient IP shocks	days	~0.3	Suprathermals	Less than ~1 MeV

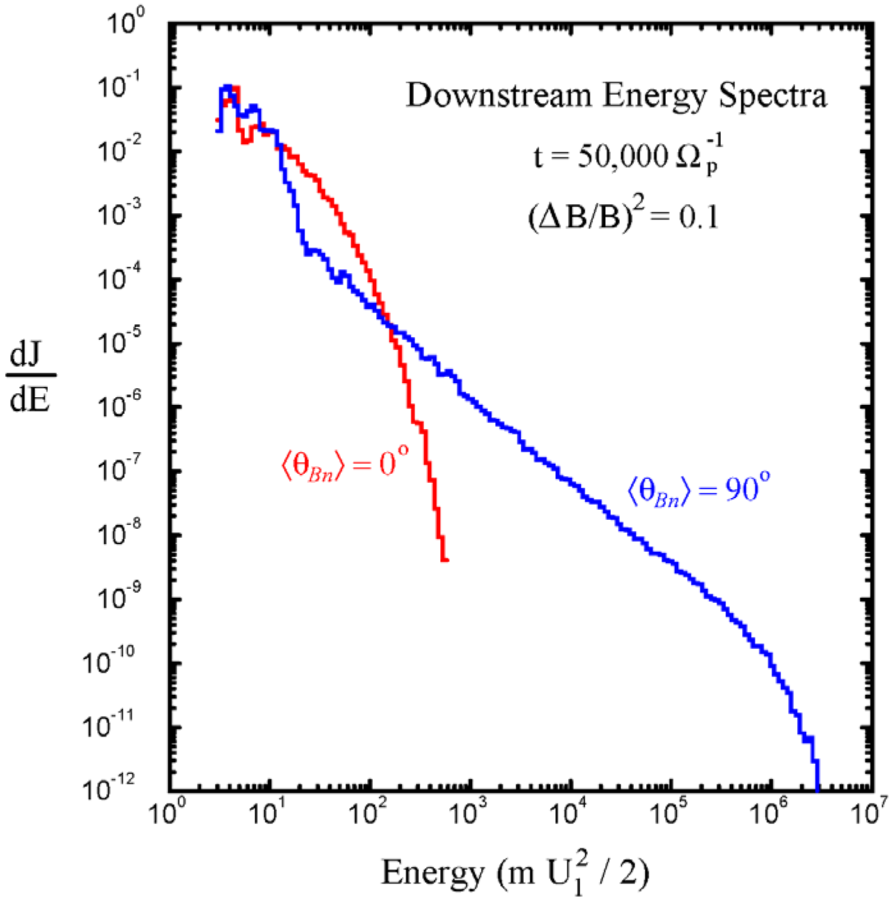
*** Suprathermals pervade the heliosphere – their origin is not well understood**

Perpendicular vs. Parallel Shocks

- The acceleration time depends on the diffusion coefficient
- because $\kappa_\perp \ll \kappa_\|$, the acceleration rate is higher for perpendicular shocks
  - For a given time interval, a perpendicular shock will yield a larger maximum energy than a parallel shock.

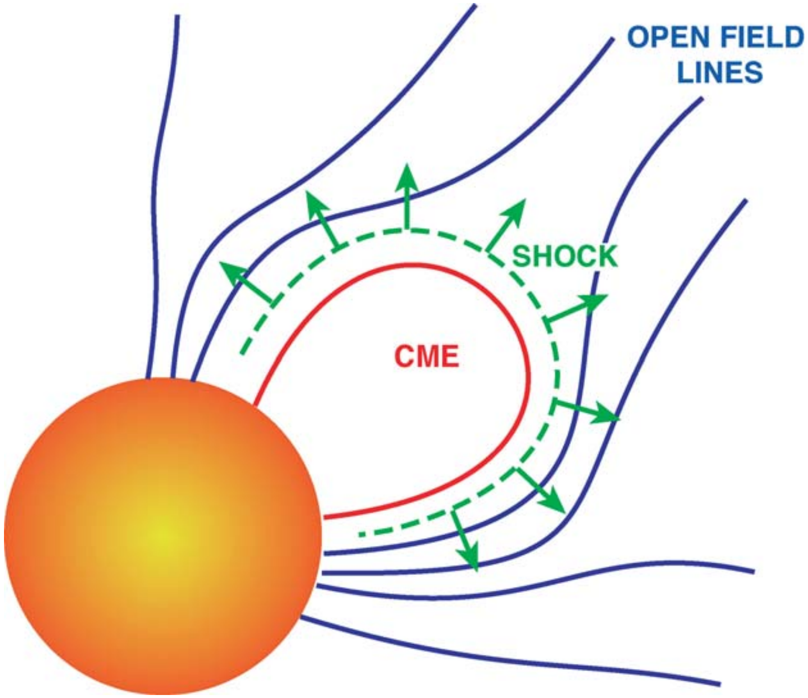
- **Perpendicular Shocks:**
  - The time scale for acceleration at a perpendicular shock is 1-2 orders of magnitude shorter (or possibly much more) than that at a parallel shock.

t = 6 minutes at 7 solar radii
(B = 0.003 Gauss)

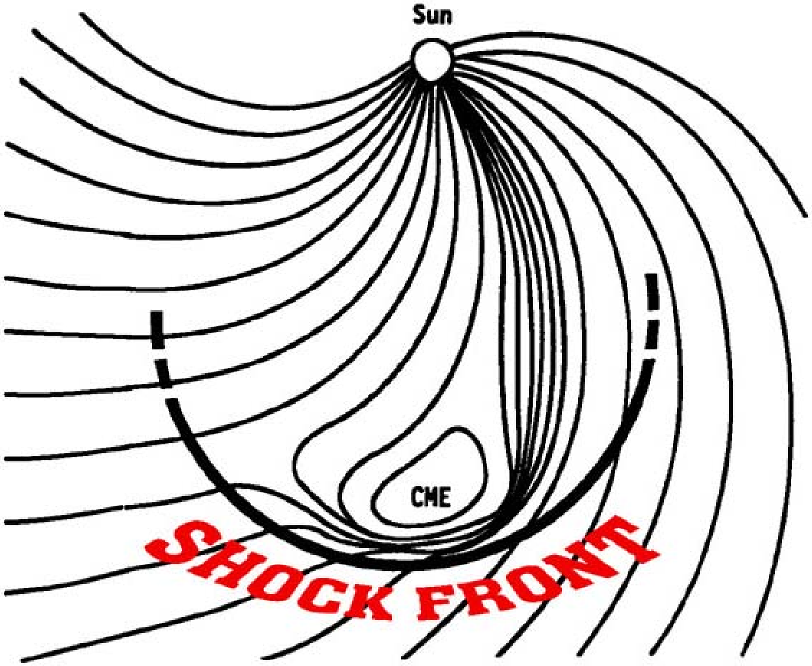


Test-particle simulations of particles encountering shocks moving in weak large-scale magnetic-field turbulence (Giacalone, 2005)

SEPs from CME-Driven Shocks



In the corona



In interplanetary space

Non-diffusive calculations

- Test-particle models (see Decker and Vlahos, 1985; Giacalone and Jokipii, 1996; Giacalone, 2005)
  - Brute force, numerical integration of the Lorentz force acting on each particle.
  - Using computers, we can integrate the trajectories of millions of particles
  - The electric and magnetic fields are specified

Non-diffusive calculations

- Self-consistent models (Giacalone et al., 1993; Giacalone, 2005)
  - “hybrid simulation” –
  - treats the electrons as a massless fluid (use the fluid equations to get the motion of the electons)
  - The ions are treated kinetically (solve Lorentz force for all ions)
  - Moments of ions are computed, and these are used to determine the fields